If You Like It, then You Shoulda Put a Sticker on It

A Model for Strategic Timing in Voting

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ABSTRACT

Sticker Voting is a voting method where ballots are cast by placing stickers on favored candidates. It differs from many other voting methods because the act of voting reveals information to other players, which induces an asymmetry of information available to subsequent voters. Voters may strategize through both the choice of the submitted ballot and the timing of its submission. In this paper, we introduce and analyze a model for strategic voter behavior in Sticker Voting. We find its equilibrium behavior and discuss how it reflects human voting behavior.

1. INTRODUCTION

Because voting is a process that takes place over time, there is an asymmetry of information that is available to earlier and later voters. The ballots cast by earlier voters inform subsequent voters. The latter may use this information to vote strategically, maximizing their chances of casting a pivotal ballot; The former may gain a first-mover advantage, establishing their favorite candidate as a lead runner by shaping what information is available to later voters. Strategic voters must decide not only which ballot to cast, but also *when* to cast their ballot.

The U.S. presidential primaries is an example of such a sequential procedure. The primaries determine each parties' presidential nominee, and are conducted as a series of elections in each state. Each state-level election determines how many delegates are sent in support of each nominee by that state. States schedule their own primary dates. The resulting elections are spread over several months. In 2016, primaries began in February and ended in June, in preparation for the November election [1]. Both parties and individual states recognize the importance of strategic timing. Certain time slots are highly prized by both the Republican and Democratic Parties. Both parties award bonus delegates to states holding their elections later in the primary season [4, 14].

Online polls are another domain which allows for strategic timing. These polls are used as a social choice mechanism for selecting anything from the cutest animal, to artistic direction for crowdfunded projects, to the winner of the Webby People's Voice award. A popular implementation of online polls is the popular group scheduling platform Doodle. Doodle allows participants to approve or decline proposed time slots. Importantly, Doodle supports *open* polls, which allow voters to view the ballots cast by previous voters before committing their own, or waiting and revisiting the poll at a later time.

In this paper, we propose the Sticker Voting framework, where voters are invited to place a sticker (their ballot) on their chosen candidate (which becomes common knowledge). In addition to the potential for casting a strategic ballot, this process also invites voters to be strategic in timing their vote. We propose a model for strategic voter behavior that incorporates strategic timing, and we analyze the strategic equilibrium in a simple Sticker Plurality Voting game. Finally, we discuss how we may use our model to capture voting behavior in the real world.

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2. RELATED WORK

In the social choice literature, Sequential Voting describes a voting process (frequently based on plurality) where voters cast their ballots in a particular order, and preceding ballots become common knowledge. This is in contrast with Sticker Voting, where the agent has a choice over the order of votes and strategic timing is possible.

Callander examines "herding" and "bandwagon" effects in Sequential Voting as an information aggregation process [5]. Voters are asked to pick from two candidates, one of which is objectively better than the other. Voters each receive a private noisy signal, and cast their ballots sequentially, based on their signal and preceding ballots. Callander shows that the resulting election begins in an informative phase, where ballots aggregate private signals of voters, but can enter a cascade phase where all subsequent voters agree on the current leading candidate. Alon et al. [2] further extend this model by giving voters intrinsic utility for having voted for the winner (in addition to selecting the better candidate). Battaglini, Morton and Palfrey [3] show that in such a game with costly votes and the possibility for abstention, early voters bear a larger cost when they choose to contribute to the information aggregation process.

Sandholm and Vulkan [15] examine bargaining games in distributed systems where agents have externally imposed deadlines. Prior to their deadline, agents may negotiate by making offers in continuous time. Interestingly, they find that the sequential equilibrium behavior for the agents is to wait until the deadline, at which point they will concede fully. This is due to the informational effect that accompanies making an early offer, which signals a weakness in bargaining position. Moreover, an accepted offer shows that the offerer has already conceded too much, and would have been better off by waiting.

Tal, Meir and Gal [16] study online human voting behavior in response to poll information. They conduct experiments on Amazon Mechanical Turk where participants are given preferences (in the form of small monetary rewards) for playing in a plurality voting game. The game may be one-shot, where poll information is fictitious; or it may be a game of Iterative Voting with other participants. Aside from a small number of erratic voters (who act randomly), most voters exercise either the "default" option (a truthful ballot in the one-shot game, or maintaining the same ballot in an iterated game), or utilized a myopic beset response.

Desmedt and Elkind [7] explore strategic behavior in Sequential Voting where voters may choose to abstain. They show how the subgame perfect Nash equilibrium may be computed, and show that when there are more than 3 candidates, the equilibrium behavior of voters are complex and sometimes counterintuitive. The outcome of the election is sensitive to the risk adversity of the voters, and the voter order.

Gaspers, Naroditskiy, Narodytska, and Walsh [8] examine the possible and necessary winner problem in Sequential Voting (which they term "social polls") when conducted in a social network setting. They find that the possible winner problem is NP-hard to compute, but propose an efficient algorithm for finding necessary winners.

Xia and Conitzer [17] study strategic behavior of agents in Sequential Voting (which they term "Stackelberg Voting"), where voter preference and voting order are public knowledge. The resulting voting game can be solved via backward induction, and may result in highly suboptimal candidates being selected.

Most relevant to our investigation of Sticker Voting is by Dekel and Piccione [6], who examine Sequential Voting where voting occurs in 2 periods, and voters are allowed to choose the period in which they wish to vote. Their model differs from ours in that, this choice must be made *prior to the election*, and prior to the realization of the voters' own preferences. Under their model, Dekel and Piccione find that all voters prefer to vote in the second period, making the sequential outcome equivalent to the simultaneous outcome.

Doodle recently emerged as a popular online poll platform for group scheduling, allowing groups to perform approval voting with open (public) or closed (private) ballots in real time. Zou, Meir and Parkes [18] examine voting behavior in over 340,000 polls. They find marked difference in voting behavior between open and closed polls. Moreover, they find that in open polls, early voters behave differently from later voters, showing evidence of strategic reasoning based on the additional information. Obraztsova, Polukarov, Rabinovich and Elkind [12] propose the Doodle Poll Game capturing this behavior, where users derive additional utility from appearing to be available. Reinecke et al. [13] have also examined how Doodle voting behavior may be affected by national culture and social norms.

3. STICKER VOTING MODEL

We consider a non-sequential voting game $\mathcal{G}$ with n voters and m candidates $\mathcal{M}$. Let $\mathcal{B}$ be the set of admissible ballots a voter may cast, and $\mathcal{B}^n$ be the set of possible ballots cast by the population of voters. Let $\mathcal{F}$ be a social choice function mapping $\mathcal{B}^n$ to the set of winners, a non-empty subset of $\mathcal{M}$. Each voter v has a private utility function $u_v : 2^{\mathcal{M}} \to \mathbb{R}$ mapping each outcome to a utility value.

We define a Sticker Voting game based on $\mathcal{G}$ by specifying a number of voting rounds $T \geq 1$. In each round, voters may cast a ballot or choose to "Wait"; this choice is made simultaneously within each round. Once a voter casts a ballot, it is committed and irreversible. Formally, in each round, each voter plays an action from the action set $\mathcal{B} \cup \{\emptyset\}$, where $\emptyset$ corresponds to "Wait" action. Once a voter casts a ballot $b \in \mathcal{B}$, their action space for subsequent rounds is reduced to the set $\{b\}$; we refer to this as moving from the **controlled game** to the **uncontrolled game**. Let $H_t \in \mathcal{B}^n$ denote the set of actions played by agents in round t. The history of play prior to current round t, $\mathcal{H}_t = (H_1, H_2, \dots H_{t-1})$ is common knowledge. The winner set is $\mathcal{F}(H_T)$, where $\emptyset$ actions are interpreted as "Abstain".

In round t, a voter may act according to a pure strategy function $\mathcal{S}$, which maps $\mathcal{H}_t$ to an action $a^t \in \mathcal{B} \cup \{\emptyset\}$. $\mathcal{S}$ maps to the action b if the agent entered the uncontrolled game by casting ballot $b \in \mathcal{B}$ in a prior round. We also allow voters to play mixed strategies, which map $\mathcal{H}_t$ to a mixed strategy, i.e. a distribution over $\mathcal{B} \cup \{\emptyset\}$.

We focus on *Markovian* strategies, where the voters do not care about the history of ballots prior to the previous round $t-1$. A Markovian strategy $\mathcal{S}$ maps t and H_{t-1} to a mixed strategy.

3.1 Plurality Sticker Voting

In this paper, we focus on the Resolute Plurality Voting Rule. Admissible ballots $\mathcal{B}$ are the candidates $\mathcal{M}$. For round t, denote the standing $\mathbf{s_t}$ as a vector whose i-th element corresponds to the number of ballots supporting candidate i in H_{t-1}, or the zero vector if $t = 1$. The social choice function $\mathcal{F}$ maps the final votes H_T to the unique candidate i with the highest s_T^i, breaking ties uniformly at random.

We consider Markovian strategies that are also anonymous to other voters. In round t, while in the controlled game, an agent's strategy simply maps t and $\mathbf{s_t}$ to a mixed strategy.

3.2 Solution Concept

The Sticker Voting Game uses the solution concept of the Perfect Bayesian Equilibrium (PBE). PBE is a refinement of Subgame Perfect Equilibrium (SPE) for sequential games. In a SPE, players act according to strategies that form a Nash equilibrium in every subgame of the original game. PBE additionally allows players to have incomplete information, where certain nodes of the game tree are indistinguishable from each other to particular players; these are called Information Sets. Players maintain beliefs corresponding to the probability that they are in a particular node in the current Information Set; their strategies are defined according to these beliefs (and may depend on the history of play). In the Sticker Voting Game, Information Sets correspond to voters not knowing the types of the other voters.

In the Plurality Sticker Voting Game, the current round and tally forms a tuple $(t, \mathbf{s_t})$ that uniquely identifies the information set for the player in the controlled game. Each information set consists of nodes representing the possible types that the remaining uncommitted voters may have. The voter has a belief over the distribution of types of the uncommitted voters.

A second set of nodes capture the uncontrolled games, with a unique node for each round t and uncontrolled tally $\mathbf{s_t}$.

4. COMPLETE INFORMATION GAME

We first consider a simplified scenario with $n = 3$ voters, $\{1, 2, 3\}$, with complete information, and $m = 3$ candidates, $\{A, B, C\}$, in a $T = 2$ round game. Player 1 has preference $A \succ B \succ C$; player 2, $B \succ C \succ A$; player 3, $C \succ A \succ B$, forming a Condorcet cycle. Each player gains utility u_1 if their favorite candidate wins, u_2 utility for their second choice, and 0 for their third choice, with $u_1 > u_2 > 0$. We also require that $2u_2 > u_1$ so that conceding to one's second place alternative is better than a three-way tie. The types of all agents are public knowledge. The following table summarizes the utilities:

Voter	A	B	C
1	u_1	u_2	0
2	0	u_1	u_2
3	u_2	0	u_1

For simplicity of notation, we denote voter v's favorite candidate as $b_{v,1}$, the second choice as $b_{v,2}$, and so on. When the v is clear from context, we omit v from the subscript. We also use $b_{v,i}$ to denote the action where v votes for i. We will actualize the utility values as $u_1 = 3$ and $u_2 = 2$.

Analysis: Final Round

Since the types are common knowledge, we use the more general solution concept of the Subgame Perfect Equilibrium, and use backward induction to solve the game. Without lost of generality, we take the perspective of Agent 1.

We begin with the final round T. If the agent is still in control, she may find the game in a number of different states:

Case 1: 2 ballots for the same candidate.

Agent 1's vote is irrelevant, and that candidate is selected

Case 2: 2 ballots for different candidates.

Agent 1 breaks ties in favor of the better option.

Case 3: 1 ballot for A

Agent 1 also votes A and gets A as the outcome.

Case 4: 1 ballot for B

Note that this ballot must be cast by Agent 2, since Agent 3 would never vote for B. In this scenario, we can break down the utilities for the remaining players in the following table. Entries indicate the winning candidate, with the payoff for the row and column players in parentheses.

		Agent 3	
		C	A
Agent 1	A	$tie(5/3, 5/3)$	$A(3, 2)$
	B	$B(2, 0)$	$B(2, 0)$

It is clear both agents will **coordinate on action** $b_{1,1} = b_{3,2} = A$ as other actions are strictly dominated, and we may iteratively remove dominated strategies.

Case 5: Only Agent 3 has voted, for $C = b_{1,3}$

We also break down utilities here:

		Agent 2	
		B	C
Agent 1	A	$tie(5/3, 5/3)$	$C(0, 2)$
	B	$B(2, 3)$	$C(0, 2)$

By the same argument before, the two agents will coordinate on **selecting** B.

Case 6: Only Agent 2 has voted, for $C = b_{1,3}$

Since Agent 3 has not voted, this is actually **Case 1** from the perspective of Agent 3. That is, since C is Agent 3's top choice, Agent 3 will also vote C, secure it as the outcome. Agent 1's vote is irrelevant.

Case 7: No votes observed

Assuming each agent plays symmetric strategies, each outcome is equally likely, giving an expected utility of 5/3.

Interestingly, **Case 4** and **Case 5** clearly show that there is no straight forward first-mover advantage in this scenario. Any agent that is the sole voter in the initial round, and votes for b_1, will force the remaining agents to coordinate in the next round, and produce b_3 as the outcome.

Analysis: Initial Round

Using backward induction, we determine the course of play in the initial round. We assume symmetric play; that is, each player v plays action $b_{v,i}$ with probability p_i, $i = \emptyset, 1, 2$, $0 \le p_\emptyset, p_1, p_2 \le 1$ and $p_\emptyset + p_1 + p_2 = 1$. We analyze the expected utility for Agent 1 for each action.

Case 1: Agent 1 plays A

As we have established, if Agent 1 plays A and the other agents plays $\emptyset$, the other agents will coordinate to select C, yielding 0 utility for Agent 1. However, Agent 1 may potentially gain an advantage if the other agents choose not to wait. The following table shows the outcomes and their payoffs for Agent 1, based on the actions of Agents 2 and 3.

		Agent 3		
		C	A	$\emptyset$
Agent 2	B	$tie(5/3)$	$A(3)$	$A(3)$
	C	$C(0)$	$A(3)$	$C(0)$
	$\emptyset$	$C(0)$	$A(3)$	$C(0)$

The expected utility for voting b_1 in the first round is

$$E(u|b_1) = \frac{5}{3}p_1^2 + 3p_2 + 3p_\emptyset p_1 \quad (1)$$

Case 2: Agent 1 plays B

		Agent 3		
		C	A	$\emptyset$
Agent 2	B	$B(2)$	$B(2)$	$B(2)$
	C	$C(0)$	$tie(5/3)$	$C(0)$
	$\emptyset$	$B(2)$	$B(2)$	$B(2)$

The expected utility for voting b_2 in the first round is

$$E(u|b_2) = 2p_1 + 2p_\emptyset + \frac{5}{3}p_2^2 \quad (2)$$

Case 3: Agent 1 plays $\emptyset$

		Agent 3		
		C	A	$\emptyset$
Agent 2	B	$B(2)$	$A(3)$	$A(3)$
	C	$C(0)$	$A(3)$	$C(0)$
	$\emptyset$	$B(2)$	$A(3)$	$*(5/3)$

The expected utility for Waiting in the first round is

$$E(u|b_\emptyset) = 3p_2 + 2p_1^2 + 5p_\emptyset p_1 + \frac{5}{3}p_\emptyset^2 \quad (3)$$

Notice immediately that even when factoring in the possibility of multiple agents voting in the initial round, Waiting dominates voting A. So we conclude that $p_1 = 0$.

Suppose we are at a symmetric mixed Nash Equilibrium, then Agent 1 must be ambivalent over the actions in its support (i.e. b_2 and $\emptyset$). So we may set equations (2) and (3) equal, and solve.

Surprisingly, the symmetric mixed Nash Equilibrium strategy for the initial round is for each agent to **play b_2 with probability 0.2, and Wait with probability 0.8**.

4.1 Rational Voter Behavior

In this simple, complete information game, rational voters will never vote for their top choice in the first round. Instead, they will vote b_2 with probability 0.2, or otherwise Wait in the first round. In the latter case, Agent 1 will vote for her favorite candidate in the second round, unless both other voters have committed their ballots and she must break a tie in her favor; or Agent 3 casts the only ballot and has voted for C, in which case Agent 1 votes for B.

5. INCOMPLETE INFORMATION GAME

Next, we consider an incomplete information scenario based on the simple game above. As before, we have $n = 3$ voters $\{1, 2, 3\}$ and $m = 3$ alternatives $\{A, B, C\}$. Players may be one of three types: Type A players have preference $A \succ B \succ C$; Type B, $B \succ C \succ A$; and Type C, $C \succ A \succ B$. The possible types form a Condorcet cycle, but there is no guarantee that such a cycle will exist in a particular realization of types. Nature assigns a type to each player with equal probability. Players know their own types, but do not know the types of other players. The game will be played over $T \geq 2$ rounds. We impose the same utility structure as before.

Analysis: Final Round T

WLOG, we consider the game from the perspective of Agent 1, who is Type A. If we are in the final round of the controlled game, with tally $\mathbf{s_t}$, let the voters' strategy $\mathcal{S}(t, \mathbf{s_t})$ be a mixed strategy playing b_i with probability $p_i^{t,\mathbf{s_t}}$, where $i \in \{1, 2, 3, \emptyset\}$. We will omit the t and/or $\mathbf{s_t}$ from the superscript where it is clear from context. Additionally, because voter strategies are symmetric with respect to type, we adopt the notational convenience of permuting the vector $\mathbf{s_t}$ so that its i-th entry corresponds to the tally of the voter's i-th favorite candidate.

Playing b_3 is strictly dominated, so by the iterated removal of dominated strategies, $p_3 = 0$ in all situations. Moreover, since this is the final round, Waiting is strictly dominated by voting b_1, so $p_\emptyset = 0$. Therefore, for any particular $\mathbf{s}$, $p_1^{\mathbf{s}} + p_2^{\mathbf{s}} = 1$. All probability values are bounded within $[0, 1]$.

Case 1: 2 ballots for the same alternative.

Agent 1's vote is irrelevant, at that alternative is selected. There are three outcomes, with utilities for Agent 1 being 3, 2 or 0.

Case 2: 2 ballots for different alternatives.

Agent 1 breaks ties in favor of the better option. There are 6 outcomes here. Agent 1 may break the tie to gain her top choice in 4 cases, and get her second choice in 2 cases.

Case 3: Agent 3 (WLOG) casts the only vote, for $A = b_{1,1}$

Agent 1 also votes A and gets A as the outcome.

Case 4: Agent 3 casts the only vote, for $B = b_{1,2}$

Agent 2 may be of one of three types. If Agent 2 is Type B, then they will also vote for B. Agent 1's vote is irrelevant, and gets a payoff of 2. The following tables break down the utility of Agent 1's actions for the other two cases:

Utility breakdown if Agent 2 is Type A

		Agent 2	
		A	B
Agent 1	A	$A(3,3)$	$B(2,2)$
	B	$B(2,2)$	$B(2,2)$

Utility breakdown if Agent 2 is Type C

		Agent 2	
		C	A
Agent 1	A	$tie(5/3, 5/3)$	$A(3,2)$
	B	$B(2,0)$	$B(2,0)$

Since Agent 2's type is not known to Agent 1, neither action is dominant. But we can calculate the expected utility for each action.

$$\begin{aligned} E(u|b_1^{(0,1,0)}) &= \frac{1}{3}(3p_1^{(0,1,0)} + 2p_2^{(0,1,0)}) \\ &+ \frac{1}{3}(\frac{5}{3}p_1^{(0,0,1)} + 3p_2^{(0,0,1)}) \\ &+ \frac{1}{3}(2) \quad (4) \\ E(u|b_2^{(0,1,0)}) &= 2 \quad (5) \end{aligned}$$

If there is a mixed equilibrium, then Agent 1 will be ambivalent over the two choices. We set equations (4) = (5), and solve to obtain

$$p_1^{(0,1,0)} = \frac{4}{3}p_1^{(0,0,1)} - 1 \quad (6)$$

We set aside this equation, and carry it forward to Case 5.

Case 5: Agent 3 casts the only vote, for $C = b_{1,3}$

Agent 2 may be of one of three types. If Agent 2 is Type C, then they will vote for C and Agent 1's action is irrelevant, and they get utility 0. The following tables break down the utility of Agent 1's actions for the other two cases:

Utility breakdown if Agent 2 is Type A

		Agent 2	
		A	B
Agent 1	A	$A(3,3)$	$tie(5/3,5/3)$
	B	$tie(5/3,5/3)$	$B(2,2)$

Utility breakdown if Agent 2 is Type B

		Agent 2	
		B	C
Agent 1	A	$tie(5/3,5/3)$	$C(0,2)$
	B	$B(2,3)$	$C(0,2)$

Since Agent 2's type is not known to Agent 1, neither action is dominant. But we can calculate the expected utility for each action. As before, if we are at a mixed equilibrium, we set the two expected utilities and solve to obtain

$$5p_1^{(0,0,1)} - 1 - p_1^{(0,1,0)} = 0 \quad (7)$$

More over, we can substitute equation (6) into (7) to obtain $p_1^{(0,0,1)} = 0$. But substituting this result back into Equation 6, we get $p_1^{(0,1,0)} = -1$. A contradiction. So we are not at a mixed equilibrium.

We then consider the pure strategy outcomes based on the actions in Case 4 and Case 5. An agent who observes $(0,1,0)$ may play $p_1^{(0,1,0)} = 1$ or $p_2^{(0,1,0)} = 1$. In addition, an agent who observes $(0,0,1)$ has options $p_1^{(0,0,1)} = 1$ or $p_2^{(0,0,1)} = 1$. There are four possible pure strategy combinations, and we may calculate the expected payoff for each player, in each scenario. For example, consider $p_1^{(0,1,0)} = 1$ and $p_1^{(0,0,1)} = 1$, where both players will play b_1 regardless of their observation. That means, if Agent 1 observed $(0,1,0)$, we will reach one of three possible outcomes: we elect A, B or reach a Tie. Thus, the expected utility will be 20/9. We repeat these calculations to formulate the outcomes in the matrix below:[1]

		Observes (0,0,1)	
		$p_1^{(0,0,1)} = 1$	$p_2^{(0,0,1)} = 1$
Observes (0,1,0)	$p_1^{(0,1,0)} = 1$	$(\frac{20}{9}, \frac{14}{9})$	$(\frac{8}{3}, \frac{4}{3})$
	$p_2^{(0,1,0)} = 1$	$(2, 1)$	$(2, \frac{2}{3})$

Notice three of the pure strategies are dominated, leaving only the top left cell as the unique symmetric Nash Equilibrium for the final round. This corresponds to the actions of **voting for the top choice regardless of the nature of the single ballot observed**. This nets an expected utility of $\frac{20}{9}$ if Agent 1 observed a ballot for her second choice, and $\frac{14}{9}$, for her third choice.

Case 6: No agent has cast any ballots, in which case Agent 1's best response is to vote honestly and hope for the best: $p_1^{(0,0,0)} = 1$, with probability $\frac{5}{9}$ of electing A, $\frac{2}{9}$ of getting a tie, $\frac{1}{9}$ of getting B, and $\frac{1}{9}$ of getting C. This results in an expected utility of $\frac{61}{27}$.

[1] While this matrix resembles a normal form game, it is only analogous to one. The rows and columns represent information states that the players find themselves in, and the actions they may take. The cell represents the payoff to the player for a particular pure strategy the agents symmetrically pursue.

Analysis: Preceding Round t

Now that we have an equilibrium analysis of the last round, we extend our analysis to preceding rounds via backward induction. Here, each agent has three actions, and Waiting is not a clearly dominated action: $p_1^{(0,0,1)} + p_2^{(0,0,1)} + p_\emptyset^{(0,0,1)} = 1$, and $p_1^{(0,1,0)} + p_2^{(0,1,0)} + p_\emptyset^{(0,1,0)} = 1$.

If Agent 1 takes the Wait action $\emptyset$, she proceeds into the information state $(t+1, \mathbf{s}^+)$ of the controlled game, where $\mathbf{s}^+$ is obtained from $\mathbf{s_t}$ by adding a number of ballots up to an including the number of uncommitted voters, representing new ballots cast this turn by the other voters. If Agent 1 casts a ballot b, then she enters into the uncontrolled game $(t+1, \mathbf{s}^+)$ (see Appendix B).

Case 1: 2 ballots for the same alternative.

Agent 1's vote is irrelevant.

Case 2: 2 ballots for different alternatives.

Agent 1 breaks ties in favor of the better option.

Case 3: Agent 3 (WLOG) casts the only vote, for $A = b_{1,1}$

Agent 1 also votes A and gets A as the outcome.

Case 4: Agent 3 casts the only vote, for $B = b_{1,2}$

As before, we may lay out the possible actions of each agent, based on the possible types of Agent 2 (recall if Agent 2 is type B, the outcome is decided regardless of the actions Agent 1):

Utility breakdown if Agent 2 is Type A

		Agent 2		
		A	B	$\emptyset$
Agent 1	A	$A(3,3)$	$B(2,2)$	$A(3,3)$
	B	$B(2,2)$	$B(2,2)$	$B(2,2)$
	$\emptyset$	$A(3,3)$	$B(2,2)$	$*(H,H)$

Utility breakdown if Agent 2 is Type C

		Agent 2		
		C	A	$\emptyset$
Agent 1	A	$tie(5/3,5/3)$	$A(3,2)$	$A(3,2)$
	B	$B(2,0)$	$B(2,0)$	$B(2,0)$
	$\emptyset$	$B(2,0)$	$A(3,2)$	$*(H,L)$

Importantly, the outcome designated as $*$ represents the outcome computed in the inductive step for the next round, where the expected utility for a player who observes a ballot for her second choice is H, or is L if a ballot for her last choice is observed ($H > L$, and $H > 2$). If the current round is $t = T-1$, then $H = \frac{20}{9}$ and $L = \frac{14}{9}$.

As before, we can write equations for expected utilities and solve to show that $b_2^{(0,1,0)}$ is dominated by $b_\emptyset^{(0,1,0)}$, if $H \geq 2$. We solve the remaining equalities in conjunction with Case 5 below.

Case 5: Agent 3 casts the only vote, for $C = b_{1,3}$

Agent 2 may be of one of three types. If Agent 2 is Type C, then they will vote for C and Agent 1's action is irrelevant, and they get utility 0. The following tables break down the utility of Agent 1's actions for the other two cases:

Utility breakdown if Agent 2 is Type A

		Agent 2		
		A	B	$\emptyset$
Agent 1	A	$A(3,3)$	$tie(5/3,5/3)$	$A(3,3)$
	B	$tie(5/3,5/3)$	$B(2,2)$	$B(2,2)$
	$\emptyset$	$A(3,3)$	$B(2,2)$	$*(L,L)$

Utility breakdown if Agent 2 is Type B

		Agent 2		
		B	C	$\emptyset$
Agent 1	A	$tie(5/3,5/3)$	$C(0,2)$	$C(0,2)$
	B	$B(2,3)$	$C(0,2)$	$B(2,3)$
	$\emptyset$	$B(2,3)$	$C(0,2)$	$*(L,H)$

We formulate expected utilities as before. We utilize Gambit [9] to solve this subgame for the $t = T-1$ case, and find that $p_2^{(0,0,1)} = 0$. Using this information (see Appendix A for details), we may solve the system of equations exactly to obtain

$$p_\emptyset^{(0,1,0)} = -\frac{3H-3L-1}{24H-3L-71} \quad (8)$$

$$p_\emptyset^{(0,0,1)} = \frac{3H-3L-8}{24H-3L-71} \quad (9)$$

and expected utilities

$$E(u|b_\emptyset^{(0,1,0)}) = \frac{4(41H-6L-121)}{3(24H-3L-71)} \quad (10)$$

$$E(u|b_\emptyset^{(0,0,1)}) = \frac{117H-19L-333}{3(24H-3L-71)} \quad (11)$$

In particular, for $t = T-1$ of the controlled game, when observing $(0,1,0)$, **Agent 1 should vote b_1 with probability $p_1^{(0,1,0)} = 64/67$ (and Wait otherwise) for an expected utility of 2.34. When observing $(0,0,1)$, she should vote b_1 with probability $p_1^{(0,0,1)} = 49/67$ for an expected utility of 1.53**.

Case 6: No ballots observed.

If no ballots are observed, all agents are in the same information set, and we may assume they act symmetrically. We denote the probability that they play their top choice, second choice and Wait as p_1, p_2, and $p_\emptyset$, respectively.

If Agent 1 Waits, then with probability $p_\emptyset^2$, we enter the next round with the tally $(0,0,0)$, which gives an expected utility of N ($N = \frac{61}{27}$ in round $T-1$). With probability $2p_\emptyset(1-p_\emptyset)$, we enter the next round with one other ballot cast (uniformly randomly selected between the candidates); each of these outcomes gives an expected utility of 3, H, and L. Finally, with probability $(1-p_\emptyset)^2$, both other agents cast their ballots. There are 9 possible outcomes (all equally likely); Agent 1 gains her top choice in 5 cases, her second choice in 3 cases, and her last choice in 1 case. This gives an expected utility of $\frac{7}{3}$. Therefore, the expected utility of waiting is

$$E(u|b_\emptyset^{(0,0,0)}) = Np_\emptyset^2 + 2p_\emptyset(1-p_\emptyset)\frac{3+H+L}{3} + (1-p_\emptyset)^2\frac{7}{3} \quad (12)$$

If Agent 1 votes for b_1, then with probability $p_\emptyset^2$, we enter the uncontrolled game $(t+1,(1,0,0))$, with expected utility U_1 (see Appendix B). With probability $2p_\emptyset(1-p_\emptyset)$, one other agent has blindly voted, resulting in the vote vector $(2,0,0)$ (utility = 3), $(1,1,0)$ (utility = $\frac{8}{3})^2$, or $(1,0,1)$ (utility = 1). Finally, with probability $(1-p_\emptyset)^2$, both other agents have blindly voted, giving a utility of $\frac{61}{27}$.

Thus, the expected utility for this action is

$$E(u|b_1^{(0,0,0)}) = U_1p_\emptyset^2 + \frac{40}{9}p_\emptyset(1-p_\emptyset) + (1-p_\emptyset)^2\frac{61}{27} \quad (13)$$

By a similar set of calculations, we get the expected utility for casting a b_2 ballot is

$$E(u|b_2^{(0,0,0)}) = U_2p_\emptyset^2 + 4p_\emptyset(1-p_\emptyset) + (1-p_\emptyset)^2\frac{49}{27} \quad (14)$$

where U_2 is the expected utility from the uncontrolled game $(t+1,(0,1,0))$, and $U_2 < U_1$. Notice $E(u|b_2^{(0,0,0)})$ is smaller than $E(u|b_1^{(0,0,0)})$ for all values of p_0. Therefore, we may assume $p_2^{(0,0,0)} = 0$, and $p_1^{(0,0,0)} + p_\emptyset^{(0,0,0)} = 1$.

Let us consider the difference of expected utility from the remaining two options:

$$E(u|b_1^{(0,0,0)}) - E(u|b_\emptyset^{(0,0,0)})$$
$$= (U_1 - N + \frac{2}{3}(H+L) - \frac{68}{27})p_\emptyset^2 + (\frac{70}{27} - \frac{2}{3}(H+L))p_\emptyset - \frac{2}{27} \quad (15)$$

Clearly, if $p_\emptyset = 0$, this would result in a negative value and $E(u|b_1^{(0,0,0)}) < E(u|b_\emptyset^{(0,0,0)})$, which is a contradiction. So we know that regardless of the values of H and L, there is a non-zero probability that an agent Waits.

If $t = T-1$, then $N = U_1 = \frac{61}{27}$ and $H + L = \frac{34}{9}$, which zeroes out the p_0^2 term, and (15) becomes $\frac{2}{27}(p_\emptyset - 1)$. Therefore, $p_\emptyset = 1$ and Agent 1 waits.

We carry forward the induction to $t = T-2$. $N = 61/27$ $U_1 = 2.1739$ and $H + L = 3.8723$. Equation 15 becomes $1/27(-2+0.2986p_\emptyset - 0.6033p_\emptyset^2)$, which is negative for all values of $p_\emptyset$. Thus, $E(u|b_1^{(0,0,0)}) < E(u|b_\emptyset^{(0,0,0)})$, and so Agent 1 waits as well. Trend continues in further rounds of induction.

Therefore, regardless of the number of rounds in the election, the rational voter always **Waits until the last round in the process before casting a sincere ballot for their top choice**. For this arrangement of candidates and voter preferences, Sticker Voting is equivalent to a simultaneous vote.

6. DISCUSSION & CONCLUSION

In our two simple instances of Sticker Voting, we observe that rational voter behavior differs dramatically. In the complete information game, voters will play a mixed strategy in

[2]Note that the Condorcet cycle is important here: if the remaining voter is Type C, she would strategically vote for A.

the first round, playing either their *second* choice or Waiting; if they chose to Wait, they will break any ties in their favor in the final round, or otherwise vote sincerely. In the incomplete information game, voters will always exercise the Wait option until they reach the final round, during which they vote sincerely.

It is interesting to contrast the two behaviors. The voters in the complete information game know that the other players are rivals, and therefore understand that there is a first-mover disadvantage if they are greedy. Yet there is also an incentive to concede early to secure acceptable compromise. In the incomplete information game, the voter is unsure as to the nature of the other players. However, more likely than not, one of the other players has the same type as her, so there is an opportunity to signal cooperation. But any incentive to do this is outweighed by the shrewdness of Waiting until the final round, where any other players with the same type as her will naturally coordinate their votes out of self interest. Additionally, in sharp contrast with the complete information game, voting second choice is never exercised as an option.

The result of our incomplete information game is in line with the results of Dekel and Piccione [6]. In their model, voters must commit to voting in one of two rounds. This decision is made prior to the election, and prior to realizing their own preferences. They find that rational voters will always vote in the second and final round. Battaglini, Morton and Palfrey [3] also remark in their work that latter voters benefit from informational effects revealed by earlier voters; while their model is fundamentally different from ours, a similar observation can be made. Finally, in Sandholm and Vulkan's bargaining game with deadlines [15], rational agents will wait until the final moment before their deadline before acting. Yet, these results appear to be at odds with the incentives offered by the Republican and Democratic Parties in the U.S., who award bonus delegates to states voting later in the primary season.

Moreover, our solution for the rational voter seem unintuitive when applied to human voters. In real world Sticker Voting venues and in online polls, we do not expect to see all (or even, a majority of) voters deliberating until the last minute to cast their ballots. We know that humans are impatient and place diminishing value on future payoffs; Are these important qualities to model in Sticker Voting? Human voters also place importance on the expressiveness of voting – they gain satisfaction from having expressed their opinion through voting sincerely. It would be interesting to conduct experiments similar to Battaglini, Morton and Palfrey [3] to elicit data on human voting behavior when using the Sticker Voting mechanism.

Additionally, we have made several assumptions about the preference structure and voter behavior for tractability of analysis. What happens when we relax these assumptions? The Condorcet cycle in the preference structure is an important element in at least one of the calculations in the model (see Footnote 2). Do the results hold if such cycle are rare in practice?

One possible model of bounded rationality that may applied to Sticker Voting is the Quantal Response Equilibrium (QRE) model [10], where players have a nonzero probability of playing each action, defined as a function of the expected payoff of that action. For instance, in the logit equilibrium (LQRE), the probability of playing an action a with expected utility $u(a|\mathbf{a_{-i}})$ where other players are using strategies $\mathbf{a_{-i}}$ is defined as

$$Pr(a|\mathbf{a_{-i}}) = \frac{e^{\lambda u(a|\mathbf{a_{-i}})}}{Z}$$

with sharpness parameter λ and normalization constant Z. QRE has also been extended to extensive form games, where the agents' future actions are treated as mixed strategies defined inductively [11].

Alternatively, it may be interesting to consider a setting where some proportion of voters are impulsive, and will commit to a ballot early in the voting process. How will the presence of such voters affect the behavior of the strategic voters? Will their actions cause a collapse in the "Waiting" equilibrium?

Finally, it would be interesting future work to investigate other models of deliberative agents in Sticker Voting setting. For instance, agents may also make use of history to infer the types of other agents, allowing them to update their beliefs of the distribution of types in population of uncommitted voters, and therefore strategize accordingly.

APPENDIX

A. UTILITIES FOR ROUND T

The expected utilities for playing b_1, b_2 or $b_\emptyset$ in round t, upon observing a single ballot for C can be calculated as follows:

$$\begin{aligned}E(u|b_1^{(0,0,1)*}) &= \frac{1}{3}(\frac{5}{3}p_1^{(0,1,0)*} + 0p_2^{(0,1,0)*} + 0p_\emptyset^{(0,1,0)*}) \\ &\quad + \frac{1}{3}(3p_1^{(0,0,1)*} + \frac{5}{3}p_2^{(0,0,1)*} + 3p_\emptyset^{(0,0,1)*}) \quad (16)\end{aligned}$$

$$\begin{aligned}E(u|b_2^{(0,0,1)*}) &= \frac{1}{3}(2p_1^{(0,1,0)*} + 0p_2^{(0,1,0)*} + 2p_\emptyset^{(0,1,0)*}) \\ &\quad + \frac{1}{3}(\frac{5}{3}p_1^{(0,0,1)*} + 2p_2^{(0,0,1)*} + 2p_\emptyset^{(0,0,1)*}) \quad (17)\end{aligned}$$

$$\begin{aligned}E(u|b_\emptyset^{(0,0,1)*}) &= \frac{1}{3}(2p_1^{(0,1,0)*} + 0p_2^{(0,1,0)*} + \frac{14}{9}p_\emptyset^{(0,1,0)*}) \\ &\quad + \frac{1}{3}(3p_1^{(0,0,1)*} + 2p_2^{(0,0,1)*} + \frac{14}{9}p_\emptyset^{(0,0,1)*}) \quad (18)\end{aligned}$$

At this point, we may use Gambit to solve the game for the $T-1$ round numerically. We get the following mixed Nash equilibrium: $p_1^{(0,1,0)} = 0.96$, $p_2^{(0,1,0)} = 0$, $p_\emptyset^{(0,1,0)} = 0.045$, and $p_1^{(0,0,1)} = 0.73$, $p_2^{(0,0,1)} = 0$, $p_\emptyset^{(0,0,1)} = 0.27$. This leads to an expected utility of 2.31 for a player who observes a ballot for her second choice, or of 1.53 for a player who observes a ballot for her last choice.

In other words, in the second-to-last round, an agent plays a mixed strategy between playing her top choice and waiting. The probability of waiting is higher if she observes a ballot supporting her last choice.

More importantly, this informs us that playing b_2 is always dominated by another strategy, when observing both $(0,1,0)$ and $(0,0,1)$. This allows us to calculate the exact solution. If we assume that $p_2^{(0,0,1)*} = 0$, we may substitute

$$p_1^{(0,0,1)} + p_\emptyset^{(0,0,1)} = 1p_1^{(0,1,0)} + p_\emptyset^{(0,1,0)} = 1 \quad (19)$$

into the previous expected utilities:

$$\begin{aligned}
E(u|b_1^{(0,1,0)*}) &= \frac{1}{3}(3) + \frac{1}{3}(\frac{5}{3}p_1^{(0,0,1)*} + 3p_\emptyset^{(0,0,1)*}) + \frac{1}{3}(2) \\
&= \frac{4}{9}p_\emptyset^{(0,0,1)*} + \frac{20}{9} \\
E(u|b_\emptyset^{(0,1,0)*}) &= \frac{1}{3}(3p_1^{(0,1,0)*} + \frac{20}{9}p_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(2p_1^{(0,0,1)*} + \frac{20}{9}p_\emptyset^{(0,0,1)*}) + \frac{1}{3}(2) \\
&= -\frac{7}{27}p_\emptyset^{(0,1,0)*}\frac{2}{27}p_\emptyset^{(0,0,1)*}) + \frac{7}{3} \\
E(u|b_1^{(0,0,1)*}) &= \frac{1}{3}(\frac{5}{3}p_1^{(0,1,0)*} + 0p_\emptyset^{(0,1,0)*}) + \frac{1}{3}(3p_1^{(0,0,1)*} + 3p_\emptyset^{(0,0,1)*}) \\
&= \frac{5}{9}p_1^{(0,1,0)*} + 1 \\
E(u|b_\emptyset^{(0,0,1)*}) &= \frac{1}{3}(2p_1^{(0,1,0)*} + \frac{14}{9}p_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(3p_1^{(0,0,1)*} + \frac{14}{9}p_\emptyset^{(0,0,1)*}) \\
&= -\frac{4}{27}p_\emptyset^{(0,1,0)*} - \frac{13}{27}p_\emptyset^{(0,0,1)*} + \frac{5}{3}
\end{aligned}$$

If we assume the equilibrium strategy is a mixed strategy comprised of the remaining actions, then we may also set $E(u|b_1^{(0,1,0)*}) = E(u|b_\emptyset^{(0,1,0)*})$, and $E(u|b_1^{(0,0,1)*}) = E(u|b_\emptyset^{(0,0,1)*})$, and solving gives us the system of equations:

$$\begin{aligned}
7p_\emptyset^{(0,1,0)*} + 10p_\emptyset^{(0,0,1)*} &= 3 \\
-11p_\emptyset^{(0,1,0)*} + 13p_\emptyset^{(0,0,1)*} &= 3
\end{aligned}$$

This solves to give us the exact solution that verifies with the empirical solution provided by Gambit, $p_\emptyset^{(0,1,0)*} = 3/67$ and $p_\emptyset^{(0,0,1)*} = 18/67$.

Using this same method allows us to compute the exact solution for any values for expected utility obtained for taking the Wait action for any given round. Let H (L) be the expected utility gained by waiting when observing $(0,1,0)$ ($(0,0,1)$), respectively. The only changes are to the utility calculations for $E(u|b_\emptyset^{(0,1,0)*})$ and $E(u|b_\emptyset^{(0,0,1)*})$ (Equation (18)), as follows:

$$\begin{aligned}
E(u|b_\emptyset^{(0,1,0)*}) &= \frac{1}{3}(3p_1^{(0,1,0)*} + 2p_2^{(0,1,0)*} + Hp_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(2p_1^{(0,0,1)*} + 3p_2^{(0,0,1)*} + Hp_\emptyset^{(0,0,1)*}) + \frac{1}{3}(2) \\
&= \frac{1}{3}(3p_1^{(0,1,0)*} + Hp_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(2p_1^{(0,0,1)*} + Hp_\emptyset^{(0,0,1)*}) + \frac{1}{3}(2) \\
&= \frac{1}{3}(3 + (H-3)p_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(2 + (H-2)p_\emptyset^{(0,0,1)*}) + \frac{1}{3}(2) \\
&= \frac{(H-3)}{3}p_\emptyset^{(0,1,0)*} \\
&+ \frac{(H-2)}{3}p_\emptyset^{(0,0,1)*} + \frac{7}{3}
\end{aligned}$$

$$\begin{aligned}
E(u|b_\emptyset^{(0,0,1)*}) &= \frac{1}{3}(2p_1^{(0,1,0)*} + 0p_2^{(0,1,0)*} + Lp_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(3p_1^{(0,0,1)*} + 2p_2^{(0,0,1)*} + Lp_\emptyset^{(0,0,1)*}) \\
&= \frac{1}{3}(2p_1^{(0,1,0)*} + Lp_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(3p_1^{(0,0,1)*} + Lp_\emptyset^{(0,0,1)*}) \\
&= \frac{1}{3}(2 + (L-2)p_\emptyset^{(0,1,0)*}) \\
&+ \frac{1}{3}(3 + (L-3)p_\emptyset^{(0,0,1)*}) \\
&= \frac{(L-2)}{3}p_\emptyset^{(0,1,0)*} + \frac{(L-3)}{3}p_\emptyset^{(0,0,1)*} + \frac{5}{3}
\end{aligned}$$

We set $E(u|b_1^{(0,1,0)*}) = E(u|b_\emptyset^{(0,1,0)*})$, and $E(u|b_1^{(0,0,1)*}) = E(u|b_\emptyset^{(0,0,1)*})$, and solve:

$$\begin{aligned}
(H-3)p_\emptyset^{(0,1,0)*} + (H - \frac{10}{3})p_\emptyset^{(0,0,1)*} &= -\frac{1}{3} \\
(L - \frac{1}{3})p_\emptyset^{(0,1,0)*} + (L-3)p_\emptyset^{(0,0,1)*} &= -\frac{1}{3}
\end{aligned}$$

which gives the solution

$$\begin{aligned}
p_\emptyset^{(0,1,0)} &= -\frac{3H - 3L - 1}{24H - 3L - 71} \\
p_\emptyset^{(0,0,1)} &= \frac{3H - 3L - 8}{24H - 3L - 71}
\end{aligned}$$

B. THE UNCONTROLLED GAME

We say Agent 1 enters the uncontrolled game node $(t{+}1, \mathbf{s})$ when she has chosen to cast a ballot in round t, resulting in the tally $\mathbf{s}$ (which includes her ballot and other ballots submitted simultaneously in round t).

In particular, we are interested in the uncontrolled game $(t{+}1, (1,0,0))$. If $t{+}1 = T$, then we know (due to symmetry) both remaining agents will vote for their top preferences. This gives an expected utility $\frac{61}{27}$ as may be expected.

However, in prior rounds $t+1 < T$, the remaining agents may be able to coordinate if they happen to vote sequentially. This only matters if the remaining agents have types B and C (a 2 in 9 chance), and depends on the probability of them waiting upon observing the controlled information state $t+1, \mathbf{s}$. As a result, the expected utility of entering this uncontrolled game is

$$E(u|(t+1,(1,0,0))) \tag{20}$$

$$= \frac{1}{27}(2p_\emptyset^{t+1,(0,1,0)}p_\emptyset^{t+1,(0,0,1)} + 8p_\emptyset^{t+1,(0,1,0)} - 10p_\emptyset^{t+1,(0,0,1)} + 61) \tag{21}$$

where $p_\emptyset^{t+1,(0,1,0)}$ and $p_\emptyset^{t+1,(0,0,1)}$ are inductively calculated for round $t{+}1$ by Equations (8) and (9). The following table shows the expected utility of entering the uncontrolled game $(t+1,(1,0,0))$, i.e. by casting a sincere ballot in round t after observing no ballots. Notice all are strictly less than $\frac{61}{27}$.

Round $t+1$	T	T-1	T-2
Utility	2.26	2.17	2.18

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