DYCOM: A Dynamic Truthful Budget Balanced Two-sided Combinatorial Market

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ABSTRACT

Recently, there has been increased attention on finding solutions for two-sided markets with strategic buying and selling agents. However, the known literature largely focuses on solutions in settings where there exists a single commodity for sale and agents ask/offer one unit of the commodity.

In this paper we present and evaluate a general solution that matches agents in a dynamic, two-sided combinatorial market. Multiple commodities, each with multiple units, are bought and sold in different bundles by agents that arrive over time.

Our solution, DYCOM, provides the first dynamic two-sided combinatorial market that allows truthful and individually-rational behavior for both buying and selling agents, keeps the market budget balanced and approximates social welfare efficiency. We experimentally examine the allocative efficiency of DYCOM under variety of distributions of bids and market demand. The experimental results are given with respect to our proven theoretical bounds and with respect to other known (dynamic and non-dynamic) two-sided markets with a single commodity as well as a non-dynamic combinatorial market. DYCOM performs well by all benchmarks and in many cases improves on previous mechanisms.

CCS Concepts

•Information systems → Web applications; •Applied computing → Electronic commerce;

Keywords

Strategic agents, Electronic commerce, combinatorial exchanges

1. INTRODUCTION

One-sided auctions have long been studied in economics and computer science. In particular, such auctions see use in the multi-agent planning domain for purposes such as task allocation [12], robot exploration [24], and resource allocation [9]. One-sided auctions aim to find high-social welfare (SWF) (an efficient) allocation of a commodity to a set of agents, while ensuring that a truthful reporting of the agents' input is their best strategy. An important extension of one-sided auctions are one-sided combinatorial auctions where multiple commodities are offered for sale. Agents bid on bundles of commodities which allows agents to express complex preferences over subsets of commodities (see [8] for many examples within). An elegant and well-studied class of combinatorial one-sided auctions are the sequential posted price auctions in which the agents are presented sequentially with a vector of prices and must choose their preferred bundle given the price vector (among the first studied are [1, 20]). One-sided combinatorial auctions have been applied to various problems, including airport time slot allocation [19], distributed query optimization [23] and transportation service procurement [22].

Recent years have brought increased attention to the problems that arise in two-sided markets, in which the set of agents is composed of buying and selling agents. As opposed to one-sided auctions where the auctioneer initially holds the commodity or the commodities and is not considered strategic, in the two-sided market the commodities are initially held by the set of selling agents, who have costs for the commodities they hold and are expected to behave strategically. The market maker's role is to match buying agents with selling agents as well as to determine what price each matched buying agent pays the market and what price the market pays each selling agent.

The cornerstone method in auction theory for high-SWF (efficient) allocation and incentivizing agents' truth-telling strategy is the Vickrey-Clarke-Groves (VCG) mechanism [25, 6, 13]. In addition to motivating agents to report their true input VCG is also individually rational (IR) in many settings. IR requires that no agent can lose by participating in the mechanism. In two-sided markets, a further important requirement is budget-balance (BB) meaning that the market does not end up with a loss. VCG is not BB except in special cases [14]. It is well known from [18] that maximizing SWF while maintaining IR and truthfulness perforce runs a deficit (is not BB) even in the bilateral trade setting, i.e., when there are just two agents trading with each other. Well-known circumventions of [18]'s impossibility in the setting of double sided auctions with a single commodity (and unit demand and supply) are [15, 16], which relax efficiency in return for maintaining the other properties of truthfulness, IR and BB. Other circumventions of [18]'s impossibility include relaxing determinism in addition to efficiency, i.e., are randomized solutions some in the simple setting of a single-commodity single-unit market [21] and some in the extended setting of combinatorial market [3]. [7, 11, 27] cir-

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cumvents [18] in the setting of single-commodity single-unit, multi-commodity single-unit and single-commodity multi-unit respectively.

The growing interest in two-sided markets is motivated by the numerous examples of applications such as stock exchanges, online advertising exchanges, pollution rights and the recent US FCC effort to reallocate electromagnetic spectrum from UHF television broadcasting to use for wireless broadband services. Many of these examples represent dynamic and uncertain environments, and thus require dynamic markets where agents arrive over time. Moreover, the examples emphasize the need for solutions that involve multiple commodities and agents that can buy and sell the multiplicity of those commodities, i.e., two-sided combinatorial markets as opposed to unit demand/supply. On the one hand due to the complex design requirements of such two-sided combinatorial markets, practical solutions for those dynamic environments such as the recent US incentive auctions circumvent the dynamic aspect of the problem by employing an iterative process [17]. And on the other hand, to our knowledge, the theoretical solutions of dynamic two-sided markets in the literature focuses on a single commodity for sale and agents ask/offer one unit of the commodity [4, 2].

Wurman et al. [26] presented a dynamic two-sided solution incentivizing truthful reporting from either the buyers or the sellers but not simultaneously from both. A different dynamic solution given by Blum et al. [2] maximizes the SWF of buyers and non-selling sellers in the single commodity unit demand setting. Finally, Bredin et al. [4] present a truthful dynamic double-sided auction that is constructed from a truthful offline double-sided auction rule also in the single commodity unit demand setting.

In this paper we present and evaluate a general solution that dynamically matches agents in a two-sided combinatorial market. Multiple commodities, each with multiple units, are bought and sold in different bundles by agents that arrive over time. Our solution, DYCOM, provides the first dynamic two-sided combinatorial market that allows truthful and IR behavior for both buying and selling agents, keeps the market BB and approximates SWF efficiency.

The main idea behind our DYCOM solution is the transformation of the two-sided combinatorial market into a one-sided combinatorial auction. The transformation of the market into an auction makes use of a novel principle: each selling agent is a buying agent of his own commodities. Thus all our dynamic market's selling agents become virtual buying agents who buy in a dynamic one-sided combinatorial auction along with our market's actual buying agents. DYCOM is a primal-dual sequential posted-price mechanism that builds upon a combinatorial auction studied in the literature [5]. However, DYCOM incorporates solutions to the design challenges imposed by the simulation process such as higher initial price constraints and payment computations for virtual buying agents. Much like other sequential posted-price mechanisms DYCOM does not require any assumption on agents' arrival order.

To validate the performance of our suggested solution, we experimentally tested the SWF efficiency of DYCOM under variety of agents' bid distributions and agents' demand against a number of benchmarks. Some of the benchmarks were dynamic and some were non-dynamic. The most notable of DYCOM's results were when compared with:

- An optimal non-dynamic and non-truthful allocation algorithm (simplex), where DYCOM's approximation approaches 0.5 of the market SWF.
- McAfee [16]'s non-dynamic single commodity unit demand market. Here DYCOM's approximation approaches 1 though DYCOM is tailored for a completely general combinatorial setting and it is dynamic unlike [16] and as such it was not expected to perform as well as [16].
- [3]'s randomized non-dynamic combinatorial market. In this comparison DYCOM's approximation approaches 10 times that of [3]'s SWF in large markets even though DYCOM is deterministic and dynamic unlike [3] and as such it was not expected to perform better than [3].

The paper's contributions are threefold. First, we provide the first dynamic two-sided combinatorial market that is truthful, IR and BB for all agents that approximates SWF efficiency. Second, our experimental tests show that our dynamic two-sided combinatorial market is a general and practical platform as it performs as well as the known McAfee [16]'s non-dynamic single-commodity unit-demand two-sided market and performs better than the randomized non-dynamic combinatorial market with limited valuations and cost domains [3]. Third, our two-sided combinatorial market transformation into a one-sided combinatorial auction is of independent interest for future work on simplifying other forms of multi-sided exchanges to the well studied form of one-sided auctions.

2. PRELIMINARIES

Consider a dynamic market model in which agents arrive over time. Agents are either buyers or sellers interested in trading multiple units of multiple commodities in bundles. Commodities are sold by selling agents and allocated to buying agents irrevocably.

Let m be the total number of non-identical commodities offered by all selling agents accumulatively. Each commodity $j \in \{1, \ldots, m\}$ has a_j identical units (or copies). Though in our model selling agents arrive dynamically we assume that a_j is a priori known to the market. The assumption that the number of a commodity's units is a priori known to the market was made by almost all previous literature on dynamic markets see ([2, 7, 26])[1]. There are practical examples where the quantity of commodities expected in the market is a priori known to the market maker. For instance consider a securities Exchange with no short sells. The number of shares of each stock issued by its company is pre-known to the exchange yet buyers and sellers arrive dynamically. Another example where the quantity of commodities expected in the market is pre-known, though without dynamic arrivals of buyers and sellers, are the newly run FCC incentive auctions where the broadcast frequencies are pre-known to the government.

A *bundle* of commodities, s, is defined as vector $(d_{s,1}...d_{s,m})$, where $0 \leq d_{s,j} \leq a_j$ is the number of units of commodity j in the bundle. We say that $s \leq s'$ if the vector of s is at most the vector of s' coordinate-wise. There are l agents who are interested in selling commodities. Each selling agent t has a bundle of commodities $S_t = (d_{S_t,1}...d_{S_t,m})$ he initially

[1]except for the work by [4] which assumed an alternative assumption of agents bounded patience.

owns and a *cost* function c_t that assigns a non-negative cost for each *bundle* $s_t \leq S_t$ of commodities, $c_t : \{0...S_{t1}\} \times ... \times \{0...S_{tm}\} \rightarrow R^+$ and any other bundle is assigned zero. We denote by c a vector of declared costs $c_1, ..., c_l$ and by c_{-t} a vector of declared costs $c_1, ..., c_{t-1}, c_{t+1}, ..., c_l$. There are n agents who are interested in buying commodities. Each buying agent i has a *valuation* function v_i that assigns a non-negative value for each *bundle* of commodities, $v_i : \{0...a_1\} \times ... \times \{0...a_m\} \rightarrow R^+$. We denote by v a vector of declared valuations $v_1, ..., v_n$ and by v_{-i} a vector of declared valuations $v_1, ..., v_{i-1}, v_{i+1}, ..., v_n$. For simplicity of notations we denote $v_i(s)$ by $v_{i,s}$ and $c_t(s)$ by $c_{t,s}$. We assume the standard assumption in combinatorial auctions literature that commodities can be perishable and the valuation function is monotonic non-decreasing. That is, for each buying agent i and $s \leq s'$, $v_{i,s} \leq v_{i,s'}$ and for each selling agent t and $s \leq s'$, $c_{t,s} \leq c_{t,s'}$. Also, for any i, $v_i(\emptyset) = 0$ and for any t, $c_t(\emptyset) = 0$ (normalization).

Bundle s is denoted as *feasible bundle* for buying agent i (selling agent t) if there is no bundle $s' \leq s$, $v_{i,s'} = v_{i,s}$ ($c_{t,s'} = c_{i,s}$). Intuitively, bundles that are not feasible contain commodities that are perishable. Let $S(i)$ ($S(t)$) be the set of feasible bundles for buying agent i (selling agent t). We assume that there are known bounds $1 < \Theta \leq \min_{\{i,j,\ s_i \in S(i), s_t \in S(t)\}} \left\{ \frac{a_j}{d_{s_t,j}}, \frac{a_j}{d_{s_i,j}} \right\}$ and $\theta \geq \max_{\{i,j,\ s_i \in S(i), s_t \in S(t)\}} \left\{ \frac{a_j}{d_{s_t,j}}, \frac{a_j}{d_{s_i,j}} \right\}$. That is, for each bundle $s_i \in S(i)$ and $s_t \in S(t)$ and commodity j, the number of commodities j in the bundle, $d_{s_i,j}$ ($d_{s_t,j}$), is at least $1/\theta$ and at most $1/\Theta$ fraction of the total number of commodities j, a_j. The θ, Θ demand bounds are parameters in our SWF approximation ratio, as will be shown in section 3. The SWF approximation ratio improves as the agents' demand decreases relative to the supply of commodities in the market. Intuitively the effect of the above parameters can be understood as improving the algorithm's performance when each participant represents a bounded share of the demand in the market. Accordingly, the algorithm performs better for large markets than thin markets as will be seen in section 4. This characteristic makes the algorithm practical and desirable for use in large markets.

Our agents are assumed to have a demand (supply) oracle representation of their valuations (costs) (a common assumption in the combinatorial auction literature e.g. [10] for valuation oracle).

Definition 2.1. *(demand (supply) oracle) For every buying agent i (selling agent t), a demand oracle for valuation (cost) v (c) accepts a vector of commodity prices* $(p_i^{(1)}...p_i^{(m)})$ *as input and outputs the demand for (supply of) the commodities at these prices, i.e. it outputs the vector* $(d_{s,1}, ..., d_{s,m})$, $s \in S(i)$ $(s \in S(t))$ *that maximizes i's utility* $\max_{s, s \in S(i)} v_{i,s} - \sum_{j=1}^{m} d_{s,j} p_i^{(j)}$ *(t's utility* $\max_{s, s \in S(t)} \sum_{j=1}^{m} d_{s,j} p_t^{(j)} - c_{t,s}$*).*

In a concrete market implementation the valuations (costs) will be given in some "bidding language" and our market will operate in polynomial time as long as the bidding language allows polynomial-time computation of answers to demand (supply) oracle queries. Note that these types of oracle queries can be easily answered for the case where each agent puts forward an arbitrary list of mutually exclusive bids for packages.

Let $A = \sum_{j=1}^{m} a_j$ be the total number of commodities. Let $s_{\max} = \max_{i,t,(s \in S(i) \wedge s \in S(t))} \{\sum_{j=1}^{m} d_{s,j}\}$ be the largest bundle requested (offered) in the market, note that, $s_{\max} \leq A$ and that we do not assume that $s_{\max}$ is pre-known.

An allocation for a two-sided market can be represented as a pair of vectors $(X, Y) = ((X_1, ..., X_n), (Y_1, ..., Y_l))$ such that the sum of the union of $X_1, ..., X_n, Y_1, ...Y_l$ is A, and $X_1, ...X_n, Y_1, ..., Y_l$ are mutually non-intersecting. The goal of the market maker is to dynamically match the agents such that each buying agent i interested in buying a bundle is allocated with available commodities of selling agents t, so as to maximize $\sum_{i=1}^{n} v_i(X_i) + \sum_{t=1}^{l} c_t(\cup s \in S(t) \setminus Y_t)$. This goal is referred to as SWF or efficiency (of trading buyers and remaining commodities).

We transform the two-sided combinatorial market into a one-sided combinatorial auction where selling agents are reduced to virtual buying agents of their own offered commodities. The one-sided combinatorial auction used to host the two-sided combinatorial market is inspired by [5]'s primal dual combinatorial auction. The goal of the auctioneer in the one-sided combinatorial auction is to partition the available commodities by allocating each buying agent i a bundle s_i, so as to maximize $\sum_{i=1}^{n} v_i(s_i)$. This goal is referred to as maximizing SWF (or efficiency).

We say that a mechanism is *truthful* if reporting the true value and cost is a dominant strategy for each agent regardless of the other agents' reports.

We say that a mechanism is *individually rational (IR)* if no agent can receive a negative utility by participating.

We say that a market is *budget balanced (BB)* if the sum of the prices paid by the buying agents is at least as high as the sum of the prices paid to the selling agents.

2.1 The one-sided combinatorial auction formulation as a linear programming problem

Our proposed one-sided combinatorial mechanism is based on solving a linear relaxation of the problem in a dynamic fashion. Let us first introduce an integer formulation for the one-sided combinatorial mechanism problem.

Let $y_{i,s} \in \{0, 1\}$ be a variable indicating that bundle s is allocated to buying agent i. Constraint (1) suggests that each buying agent is allocated at most one bundle. Constraint (2) suggests that the number of units sold from commodity j is at most a_j.

We relax the integrality constraints $y_{i,s} \in \{0, 1\}$ in order to achieve the below linear program formulation that upper bounds the maximum SWF.

$$[Dual] \qquad \max \sum_{i=1}^{n} \sum_{s \in S(i)} v_{i,s} y_{i,s}$$

$$\sum_{s \in S(i)} y_{i,s} \leq 1 \forall 1 \leq i \leq n \qquad (1)$$

$$\frac{1}{a_j} \sum_{i=1}^{n} \sum_{s \in S(i)} d_{s,j} y_{i,s} \leq 1 \forall 1 \leq j \leq m \qquad (2)$$

$$y_{i,s} \geq 0 \forall 1 \leq i \leq n, s \in S(i)$$

Note that the number of variables may be exponential. However, our algorithm never solves this linear formulation explicitly. We refer to this formulation as the dual program [$Dual$]. To obtain the corresponding primal program [$Primal$], we define variable z_i for each buying agent i, and variable x_j for each commodity j. The primal linear formulation is as follows.

$$[Primal] \qquad \min \sum_{i=1}^{n} z_i + \sum_{j=1}^{m} x_j$$
$$z_i + \sum_{j=1}^{m} \frac{d_{s,j}}{a_j} x_j \geq v_{i,s} \forall 1 \leq i \leq n, s \in S(i) \quad (3)$$
$$z_i, x_j \geq 0 \forall 1 \leq i \leq n, 1 \leq j \leq m$$

Note that the dual problem described above is not the integer formulation of the traditional combinatorial auction problem but rather a linear programming formulation that upper bounds the maximum social welfare and is never solved explicitly by the solution we present in the paper. Our solution solves the primal problem presented and therefore it is presented as primal.

3. DYCOM AND THE SIMULATION OF TWO-SIDED COMBINATORIAL MARKET AS A ONE-SIDED

In this section we first discuss how to transform a two-sided combinatorial market into a one-sided combinatorial auction such that one can conclude the allocation and prices of the buying agents as well as the allocation of the selling agents and the payments they receive. We then present DYCOM and prove its economic properties and approximation.

Consider a dynamic market in which agents arrive over time and prices increase with demand (we make the common assumption in online mechanism design literature that the order of arrival is arbitrary and agents have no control over it. This assumption can also be found in [1], [5] and many citations within.). Agents are either buyers or sellers which arrive once and are faced with a vector of prices. Agents can demand/supply a bundle of their choice in the given prices immediately or leave permanently. Selling agents that supply a bundle stay at the market until their supply is sold (or return to them in market closing time). For every arriving agent t which is interested in selling bundles of commodities $S(t)$ and initially owns commodities S_t, we construct a virtual agent i that is interested in buying some of selling agent t's commodities. In order to simulate a virtual buying agent i that represents selling agent t's interests we need to allow virtual agent i to buy the commodities that are not beneficial for selling agent t to sell. For example if selling agent t has one unit of commodity 1 and one unit of commodity 2, his cost function is $c_{t,\{1\}} = 10$, $c_{t,\{2\}} = 5$, $c_{t,\{1,2\}} = 14$ and he is presented with prices $p_t^{(1)} = 8$, $p_t^{(2)} = 7$ then he is not interested in selling commodity 1. Therefore the virtual buying agent that represents him will buy commodity 1. Since all that we have access to is the agent's demand(supply) oracle, in order to simulate selling agent t as a buying agent of t's commodities we query each selling agent t's supply oracle as he arrives and allocate the created virtual buying agent with the commodities $S_t \setminus s_t$ where s_t is the bundle answered by t's supply oracle. Selling agent t's commodities that were not bought by its virtual buying agent are offered to the "regular" (non virtual) buying agents that arrive in the time periods that follow.

We assume a priori knowledge of the values $v_{\max}$ and $c_{\min}$ such that $v_{\max} \geq \max_{i,s}\{v_{i,s}\}$, $c_{\min} \leq \min_{t,s}\{c_{t,s}\}$ and $v_{\max} > c_{\min}$. It is easy to verify that $v_{\max}$ and $c_{\min}$ knowledge is necessary in order to obtain non-trivial approximation ratio. First we consider the a priori knowledge of $v_{\max}$. If $v_{\max}$ is unknown to the algorithm, then any deterministic algorithm has an unbounded efficiency approximation ratio even if there is only a single commodity (with multiple units). To see this, consider selling agents with cost zero for all commodities and consider the following simple adversarial sequence. In each iteration the next buying agent would like a single unit of the (single) commodity and his bid is the smallest value of the remaining buying agents that still need to arrive. If there is no such value then certainly the algorithm has no bounded efficiency. Otherwise, the algorithm always allocates all units, and after allocating all units then, the next buying agent has value that is very large compared to all previous bids. Similar argument can be made for the necessity of $c_{\min}$.

As the assumption of a priori knowledge of $v_{\max}$ and $c_{\min}$ is necessary in order to obtain a non-trivial approximation all previous literature on dynamic markets even ones with single commodity assume similar a priori knowledge of the max, min values (see [2, 7, 26]). The only previous work on dynamic markets that does not assume a similar assumption to the max, min values, is the work by [4]. However [4]'s work assumes an alternative assumption: agents bounded patience, that without it no reasonable efficiency can be achieved.

Let $s^i_{\max} = \max_{s|y_{i,s}=1}\{\sum_{j=1}^{m} d_{s,j}\}$ be the maximal size of any bundle allocated by DYCOM until agent i's arrival (including i) and let $\psi = \frac{\ln(1+s^i_{\max}(v_{\max}-c_{\min}))}{1-1/\Theta}$.

DYCOM is composed of initialization stage (the first two for loops) and a running loop that handles dynamically arriving agents. The loop for the arriving agents has 5 steps.

- Step (1) update the prices of all commodities for the new agent that arrived.
- Step (2) query the arriving agent for his demand or supply (depending on the type of agent) of commodities given the current prices.
- Step (3) handle selling agents by converting each arriving selling agent in to a virtual buying agent. The virtual buying agent is configured to buy the commodities that the selling agent is better off keeping and not selling given the current market prices, i.e., his total commodities bundle S_i minus the bundle of commodities that are most beneficial for him to sell according to his supply oracle s_i. Payment to the arriving agent is made every time his commodities are bought by future arriving buying agents. The payments are computed according to the prices that were presented to the selling agent at his arrival[2].
- Step (4) handle buying agents by allocating each arriving buying agent his requested bundle at current prices and charging him according to those prices. In this step DYCOM pays selling agents for commodities that were bought by the currently arriving buying agent.

[2]Note that the IR property is not affected by the later payments since if units of commodities in s_i are not sold in the market by its closing time then those units can be returned to seller i. Also note that similarly to [4] we could have changed our algorithm to pay the arriving selling agents instantaneously and have the market "hold" the commodities until bought, however such approach will lead to market deficits during the market run as occurs in [4]'s market.

- Step (5) updates the parameters given the new allocation and in particular updates the primal parameter x_j^i for the next arriving agent's prices. The update formula, x_j^{i+1}, is motivated by the idea that no commodity j is allocated more than a_j times. This can be achieved by increasing the price of every commodity j such that after the allocation of at least $(1 - 1/\Theta)a_j$ units of the commodity the price reaches the level of $v_{\max}$. At this high price no agent can afford to buy the commodity. Meaning, no more units of the commodity will be sold after the price reaches $v_{\max}$. As each allocation of a commodity to an agent is at most $1/\Theta$ of the commodity, no more than a_j allocations of the commodity can occur in total.

DYCOM

For each commodity j set $x_j^1 = c_{\min} a_j$, , $s_{\max}^1 = 0$

For each buying and selling agent $1 \leq i \leq n + l$
set $z_i = 0$, $y_{i,s} = 0$

For each arriving agent $i = 1 \ldots n + l$
(1)for each commodity j set the price $p_i^{(j)} = x_j^i / a_j$
(2)input current prices $p_i^{(1)} \ldots p_i^{(m)}$ to agent i's demand/supply oracle and output demand/supply bundle $d_{s,1} \ldots d_{s,m}$, $s_i \in S(i)$
(3)if i is a selling agent then construct a virtual buying agent i by:
allocating him the bundle $s = S_i \setminus s_i$
paying him the future payment determine at (4)
query i on $c_{i,s}$, set $v_{i,s} = c_{i,s}$
(4)if i is a buying agent
allocate i with bundle $s = s_i$
charge i $p_i = \sum_{j=1}^{m} d_{s,j} p_i^{(j)}$
query i on $v_{i,s}$
for $k = 1 \ldots i - 1$
for every unit of commodity j
of virtual buying agent $(i - k)$
that is allocated to agent i in $d_{s,j}$,
pay agent $(i - k)$ the price $p_{i-k}^{(j)}$
(5)Update:
$y_{i,s} = 1$, $z_i = v_{i,s}$, recompute $s_{\max}^i$
for all j: $x_j^{i+1} \leftarrow x_j^i \exp\left(\frac{d_{s,j} y_{i,s}}{a_j} \psi\right)$
$+ a_j \left(\frac{1}{s_{\max}^i} - c_{\min}\right)\left(\exp\left(\frac{d_{s,j} y_{i,s}}{a_j} \psi\right) - 1\right)$

3.1 Analysis

In this section we analyze the performance of the DYCOM solution as a truthful, IR, BB and SWF maximizing market. Our analysis first shows that the market is truthful and IR both for buying and selling agents and does not run a deficit. Then we focus on the analysis of the SWF approximation ratio.

LEMMA 3.1. *DYCOM is truthful and IR for buying and selling agents and is a BB market.*

PROOF. We start by claiming that DYCOM is truthful. Since agents have no control over their arrival order they can not affect the commodities prices they are faced with. Nevertheless agents can potentially misreport their demand/supply bundle or can misreport its value/cost. We first claim the buying agents are weakly better off reporting their true demand bundle and their true value for it. Assume for the contrary that a buying agent requested a bundle s' that is not the bundle s that was recommended to him by his demand oracle. As the demand oracle outputs the bundle that maximizes the agent's utility given the price vector, when allocated s', the agent can not gain a higher utility than s. Thus the buying agent is (weakly) better off reporting his true demand bundle. Any declaration of s's value can not change the allocation (and therefore can not change the buying agent's utility) as the allocation is determined by the bundle demand. Moreover buying agent's lie will be immediately exposed if he reports the value of s such that it is less than the total price of the bundle s as his demand oracle is utility maximizing.

We continue by claiming that the selling agents are weakly better off by reporting their true supply bundle and their true cost for the bundle. Assume to the contrary that a selling agent t requested a bundle s' that is not the bundle s that was recommended to him by his supply oracle. First assume the case that there exists a unit of commodity j that is in s however it is not in s'. That means that the unit of commodity j will be allocated to the virtual buying agent constructed of selling agent t and t will not be paid for it. However we know that given the prices presented to t and his supply oracle, his utility will increase if we will not keep the unit of commodity j and will get paid for it, in its presented price. Thus t is better of requesting bundle s. Now assume that there exists a unit of commodity j that is in s' however it is not in s. That means that the unit of commodity j will not be allocated to the virtual buying agent constructed of selling agent t and t will be paid for it. However since selling agent t's supply oracle is utility maximizing, we know that t's utility will increase (or at lease will not decrease) by not selling the unit of commodity j and not get paid for it. Thus the selling agent is (weakly) better off reporting his true supply bundle. Any declaration of s's cost can not change the allocation (and therefore can not change the selling agent's utility) as the allocation is determined by the supply of bundles. Moreover, a selling agent's lie will be immediately exposed if he reports the cost of s such that it is more than the total price of the bundle s as his supply oracle is utility maximizing.

We continue by claiming that DYCOM is IR. For buying agents DYCOM is IR since a buying agent only pays for units of commodities he is allocated and his payment is computed based on the commodities price vector presented to him. As each buying agent's demand oracle is utility maximizing no allocation will result in a negative utility for a buying agent. For selling agents DYCOM is IR since a selling agent only gets paid for units of commodities that his virtual buying agent is not allocated (which is exactly his supply bundle) and the mechanism's payment to him is computed based on the commodities price vector presented to him. As each selling agent's supply oracle is utility maximizing no sell will result in a negative utility for a selling agent.

Now we claim that DYCOM does not run a deficit, i.e., DYCOM is BB. Since selling agents get paid according to the commodities prices that are presented when they arrive and buying agents pay according to the current prices they see, and since prices are non-decreasing between arrival times, every buying agent payment on every unit of a commodity will be at least as high as the payment for its selling agent.

□

We continue by analyzing the SWF approximation ratio.

Lemma 3.2. *DYCOM approximates the SWF of the trading buying agent and the remaining commodities with in* $O(\Theta[(1+s_{\max}(v_{\max}-c_{\min}))^{\frac{1}{\Theta-1}}-1]+\theta)$.

Before we present the proof of our approximation claim we like to compare DYCOM's approximation ratio with that of the other known combinatorial two sided market by [3]. [3]'s approximation for the SWF of the trading buying agent and the remaining commodities in a randomized mechanism and if all valuations and costs are subadditive[3] is $8H_{s_{\max}}$ where $H_{s_{\max}}$ is the $s_{\max}$ harmonic number. Their mechanism assumes distributional knowledge of the median value of each selling agent's Θ, θ bounds. Figure 1 shows that for large markets DYCOM achieves better theoretical approximation ration than [3] even though [3]'s solution is randomized non-dynamic and the approximation ration is only guaranteed for the cases where valuations and costs are subadditive and not generated for the general case as ours[4].

Proof. In order to show DYCOM's SWF of the trading buying agent and the remaining commodities' approximation ratio, it is enough to show the SWF approximation ratio for buying and virtual buying trading agents as the last ones are allocated the remaining commodities.

Let $\Delta Primal$ be the change in the value of the primal solution and let $\Delta Dual$ be the change in the value of the dual solution. After each agent's arrival DYCOM updates a primal solution $[Primal]$ and a dual solution $[Dual]$. In order to show the approximation ratio we need to prove that (i) the primal solution produced by DYCOM is feasible. (ii) the dual solution output by DYCOM is feasible. (iii) after each agent arrival, $\Delta Primal$ is at most w times $\Delta Dual$, where w would have been our desired approximation ratio if both the primal and dual solutions were initially 0. In that case we would have achieved an approximation of at least $1/w$ times the feasible primal solution we produce. Since our primal solution is not initially 0 but $\sum_{j=1}^{m} c_{min}a_j$, we need to reduce $\phi=\sum_{j=1}^{m} c_{min}a_j$ from $[Primal]$ and we conclude that our approximation is at least $\frac{1}{w+\frac{\phi}{[Dual]}}$ times the feasible primal solution we produce. The lemma's claim follows directly by weak duality.

Primal feasibility: It is easy to verify that the primal solution produced by DYCOM satisfies all primal constraints. We omit the details due to space limitations.

Primal-Dual relation: We need to show the relations of $\Delta Primal$ and $\Delta Dual$ created by the arrival of agent i. Denote $\Delta x_j = x_j^{i+1}-x_j^{i}$. Let $q=\left(\exp\left(\frac{d_{s,j}}{a_j}\psi\right)-1\right)$ and let $q'=\left(\exp\left(\frac{\psi}{\Theta}\right)-1\right)$ for the ease of presentation.

$$\begin{aligned}
\Delta \quad & Primal = z_i + \sum_{j=1}^{m}\Delta x_j = v_{i,s} \\
+ \quad & \sum_{j=1}^{m}\left[x_j\left(\exp\left(\frac{d_{s,j}}{a_j}\psi\right)-1\right)+\left(\frac{a_j}{s^i_{\max}}-a_j c_{\min}\right)q\right] \\
= \quad & v_{i,s}+\sum_{j=1}^{m}\left(x_j\cdot\frac{d_{s,j}}{a_j}+\frac{d_{s,j}}{s^i_{\max}}-d_{s,j}c_{\min}\right)\cdot\frac{a_j}{d_{s,j}}q \\
\leq \quad & v_{i,s}+\sum_{j=1}^{m}\left(x_j\cdot\frac{d_{s,j}}{a_j}+\frac{d_{s,j}}{s^i_{\max}}-d_{s,j}c_{\min}\right)\cdot\Theta q' \qquad (4)\\
\leq \quad & v_{i,s}+(v_{i,s}+1-c_{\min})\cdot\Theta q' \qquad (5)\\
\leq \quad & v_{i,s}+(v_{i,s}+1)\cdot\Theta q' \qquad (6)\\
\leq \quad & \left(1+2\Theta q'\right)\Delta Dual \qquad (7)
\end{aligned}$$

Inequality (4) follows as for every $x,\psi\geq 1$ $x(e^{\frac{\psi}{x}}-1)$ is monotonic decreasing. Inequality (5) follows as $\sum_{j=1}^{m}\frac{d_{s,j}}{s^i_{\max}}\leq 1$ ($s^i_{\max}$ is the size of the maximal bundle allocated until current agent's arrival) and as $\sum_{j=1}^{m} d_{s,j}\frac{x_j}{a_j}=\sum_{j=1}^{m} d_{s,j}p_i^{(j)}\leq v_{i,s}$. Also, since the minimal size allocated bundle is a single unit of one item type then $\sum_{j=1}^{m} d_{s,j}\geq 1$. Inequality (6) follows since $c_{\min}>0$. Finally, Inequality (7) follows since $\Delta Dual = v_{i,s}$. Note that $s^i_{\max}$ and ψ are non-decreasing throughout the agents' arrivals. Last but not least is bound-

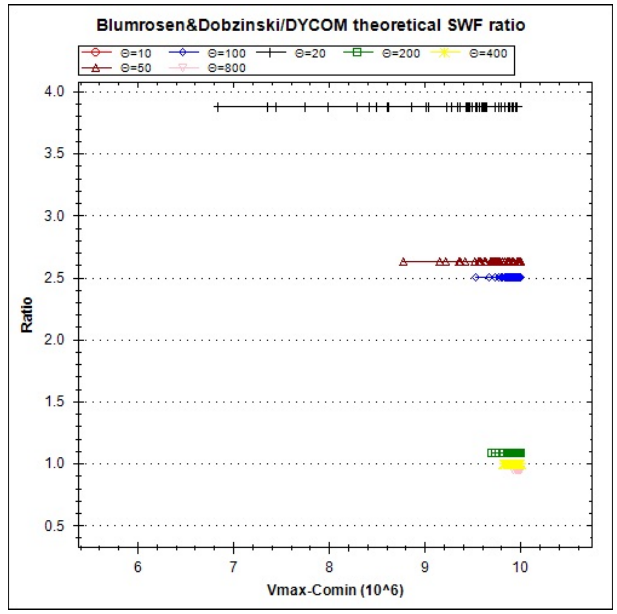

Figure 1: DYCOM's theoretical SWF approximation ratio under different bounds of agents' demand/supply level with respect to the overall supply of commodities in the market vs. Blumrosen and Dobzinski 2014 theoretical SWF approximation ratio. When demand/supply of each agent is bounded by at least 1/400 of total market units DYCOM's theoretical SWF approximation ratio is better than Blumrosen and Dobzinski 2014.

[3]Subadditive - roughly speaking, the value of bundle A plus the value of bundle B is greater than the value of their union.
[4]Figure 1 shows its finding under subadditive valuations and costs. When we generate data removing this assumption DYCOM gives even better theoretical guarantees for the presented large markets. This graph was omitted due to page limitations

ing from above $\phi/[Dual]$.

$$\begin{aligned}\frac{\phi}{[Dual]} &= \frac{\sum_{j=1}^{m} c_{\min} a_j}{\sum_{i=1}^{n} v_{i,s} y_{i,s}} \\ &\leq \frac{\sum_{j=1}^{m} c_{\min} a_j}{v_{i,s}} \leq \frac{\sum_{j=1}^{m} x_j^i}{v_{i,s}} \quad (8) \\ &\leq \frac{\sum_{j=1}^{m} \left(x_j^i \frac{d_{s,j}}{a_j} \right) \cdot \frac{a_j}{d_{s,j}}}{v_{i,s}} \\ &\leq \frac{\sum_{j=1}^{m} x_j^i \frac{d_{s,j}}{a_j} \cdot \theta}{v_{i,s}} \leq \theta \quad (9)\end{aligned}$$

Inequality (9) follows since $\sum_{j=1}^{m} x_j^i \frac{d_{s,j}}{a_j} \leq v_{i,s}$.

Substituting ψ and $s_{\max}^i$ we achieve the approximation ratio of:
$1+2\Theta q' + \theta \leq 1+2\Theta \left(\exp\left(\frac{\ln(1+s_{\max}(v_{\max}-c_{\min}))}{\Theta-1} \right) - 1 \right) + \theta = O(\Theta[(1+s_{\max}(v_{\max}-c_{\min}))^{\frac{1}{\Theta-1}} - 1] + \theta)$. Dual feasibility: DYCOM's solution is the solution produced by the dual solution. In order to prove dual feasibility we need to show that no agent is allocated more than a single bundle and that no commodity j is allocated more than a_j times. Since each arriving agent is asked to declare a bundle of interest through a demand oracle and the demand oracle outputs a single bundle as an out come, the single bundle constraint is satisfied. In order to prove the commodity constraint we prove that for every commodity j the price reaches the level of $v_{\max}$ after the allocation of at least $(1-1/\Theta)a_j$ units of the commodity. At the resulting high price no agent can afford to buy the commodity any more. Meaning, no more units of the commodity will be sold after the price reached the $v_{\max}$ level . As each allocation of a commodity for an agent is at most $1/\Theta$ of the commodity, no more than a_j allocations of the commodity can occur in total. We look for a price expression such that when agent g arrives $\frac{1}{a_j}\sum_{i=1}^{g-1}\sum_{s\in S(i)} d_{s,j} y_{i,s} \geq 1-1/\Theta$, then the price is at least $v_{\max}$. We show that the price computed by DYCOM is such. We prove by induction that the price of one unit of commodity j at the arrival time of agent g is as follows:

Let $Q = \frac{\ln(1+s_{\max}^g(v_{\max}-c_{\min}))}{(1-1/\Theta)a_j}$ for the ease of presentation and let $s_{\max}^g$ is the maximal bundle allocated by DYCA up to the arrival of agent g.

$$\begin{aligned} p_g^{(j)} &= \frac{x_j^g}{a_j} \geq \\ \frac{1}{s_{\max}^g} & \left(\exp\left(Q \cdot \sum_{i=1}^{g-1} \sum_{s\in S(i)} d_{s,j} y_{i,s} \right) - 1 \right) + c_{\min} \quad (10)\end{aligned}$$

We omit the details of the induction proof due to space limitations.

□

4. EXPERIMENTAL RESULTS

We conduct an empirical evaluation of our suggested solution's performance against a range of known market benchmarks. We compare the allocative SWF efficiency of buying agents and unallocated commodities of DYCOM with the known non-dynamic single commodity unit-demand solution by [16] (Figure 2) and the dynamic single-commodity unit-demand solution by [2] (Figure 5)[5]. We also compare the allocative SWF efficiency DYCOM's buying agents and unallocated commodities with [3]'s randomized non-dynamic combinatorial market. (Figure 6)[6]. Our experimental results show that though DYCOM is dynamic, combinatorial and more general it can perform in practice as well as the above known solutions that were tailored for limited market settings and perform even better for some of the above known solutions for large markets. While [2]'s solution's performance mainly depends on the size of the valuation/cost range in the market (which may be large in an electronic global market), our DYCOM solution performs best on large markets where no buying or selling agents control a large portion of the demand or supply (See Figure 5). As was seen in Subsection 3.1 Figure 1 DYCOM theoretically performs better than [3] in large markets where the agents' values and costs are taken over a large spread and each agent's demand/supply is bounded by at least 1/400 of total market units. Interestingly the performance gap improves favorably towards DYCOM in the practical comparison. Figure 6 shows that even if each agent's demand/supply is bounded by at least 1/200 of total market units, DYCOM performs better. Figure 6 shows its finding under subadditive valuations and costs. When we generate date removing this assumption DYCOM performs even better with respect to [3] under the same size market's demand/supply bounds. The graph was omitted due to page limitations.

All results presented were averaged over 1000 trials. The comparisons with [16, 2] were performed on a market with 800 units of a commodity. In all the experiments we found minimal to no qualitative differences between the use of different distributions. We also compared DYCOM's practical performance with that of the theoretical results for multiple commodities (Figure 3 and Figure 4) and found that in practice DYCOM's SWF approximation ratio is improved by an order of magnitude and converges to half the SWF of an optimal non-dynamic combinatorial solution (simplex). We note that the theoretical approximation ratio proven in Subsection 3.1 converges to 0.06 in the runs we performed (Figure 4).

5. CONCLUSION AND DISCUSSION

In this paper we present and evaluate DYCOM the *first dynamic two-sided combinatorial market* that allows truthful and IR behavior for both buying and selling agents, keeps the market BB and approximates SWF efficiency. DYCOM is a general solution that dynamically matches agents that arrive over time in a two-sided combinatorial market with multiple commodities of multiple units.

The main idea behind our DYCOM solution is a transformation of the two-sided combinatorial market into a one-sided combinatorial auction. The transformation of the market into an auction makes use of a novel principle that each selling agent is a buying agent of his own commodities. DYCOM is a primal-dual sequential posted-price mechanism

[5]We omitted the comparison of DYCOM with [4]'s solution as they conclude that their Chain mechanism performs essentially the same as [2]'s mechanism in practice.

[6]the comparison is done such that all valuations and costs are subadditive as [3] assumes such valuations and costs as part of their approximation ratio bound.

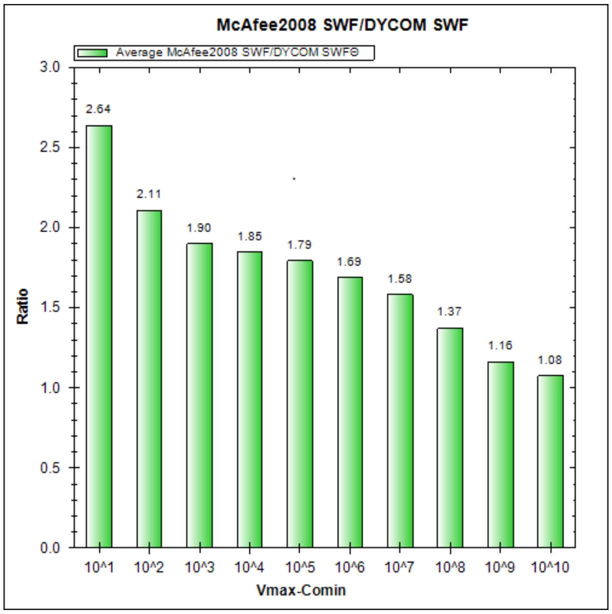

Figure 2: McAfee 2008's SWF vs. DYCOM's SWF. DYCOM seems to converge to an identical SWF approximation ration as McAfee as the valuation and cost range grows.

and its economic properties as well as its approximation guarantee are theoretically proven.

To validate the performance of our DYCOM solution, we experimentally tested the SWF efficiency of DYCOM under variety of agents' bid distributions and agents' demand against a number of benchmarks. Our experimental tests show that DYCOM is a general and practical platform as 1) it performs as well as the known McAfee [16]'s non dynamic single-commodity unit-demand two-sided market though DYCOM is tailored for a completely general combinatorial setting and it is dynamic unlike [16] and 2) it's approximation approaches 10 times that of [3]'s market's SWF in large markets though DYCOM is deterministic and dynamic unlike [3] which is randomized and non dynamic.

In addition to providing a practical solution to the important problem of a dynamic two-sided combinatorial market, we believe that our two-sided combinatorial market transformation into a one-sided combinatorial auction is of independent interest for future work on reducing other forms of multi-sided exchanges to the well studied form of one-sided auctions.

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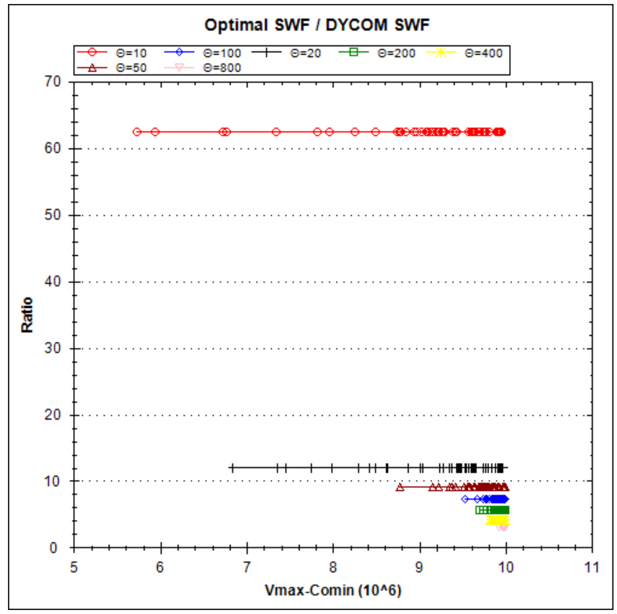

Figure 3: DYCOM's practical SWF approximation ratio under different bounds of agents' demand/supply level with respect to the overall supply of commodities in the market. The red line represents demand/supply of each agent that is bounded by 1/10 of total market units. The pink line represents demand/supply of each agent that is bounded by 1/800 of total market units. In large markets, i.e. where the demand/supply bound is smaller DYCOM's practical SWF approximation ratio converges to 0.5.

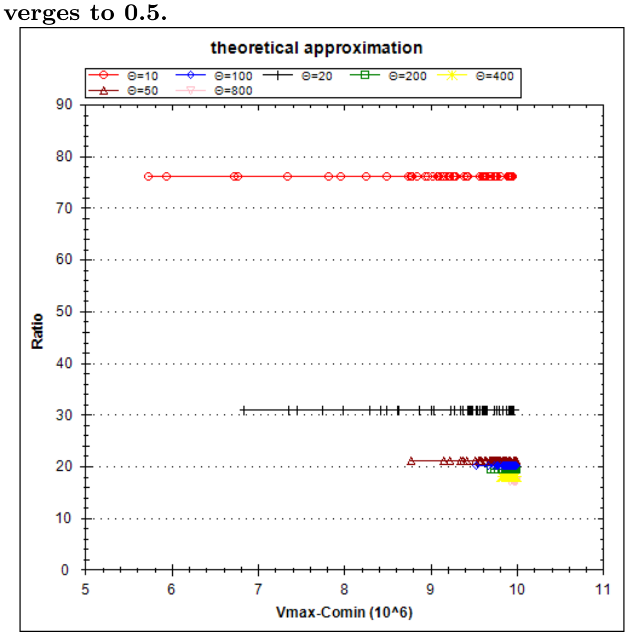

Figure 4: DYCOM's theoretical SWF approximation ratio under different bounds of agents' demand/supply level with respect to the overall supply of commodities in the market. The red line represents demand/supply of each agent that is bounded by 1/10 of total market units. The pink line represents demand/supply of each agent that is bounded by 1/800 of total market units. In large markets, i.e. where the demand/supply bound is smaller DYCOM's theoretical SWF approximation ratio converges to 0.06.

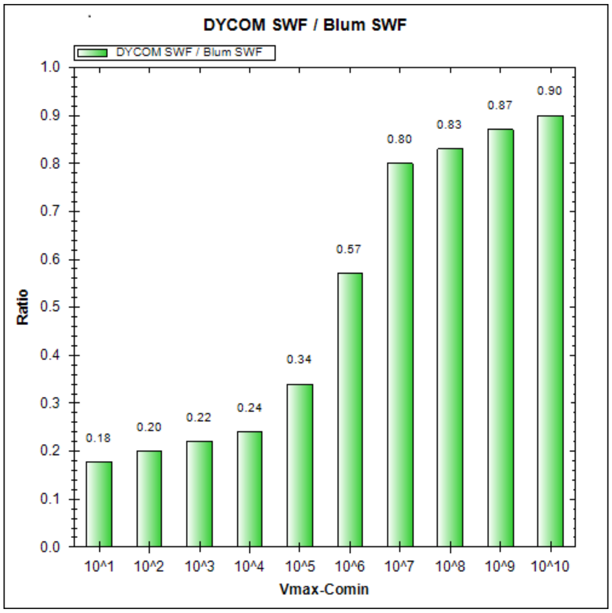

Figure 5: Blum et al.'s SWF vs. DYCOM's SWF. DYCOM seems to converge to an identical SWF approximation ration as Blum et al. as the valuation and cost range grows.

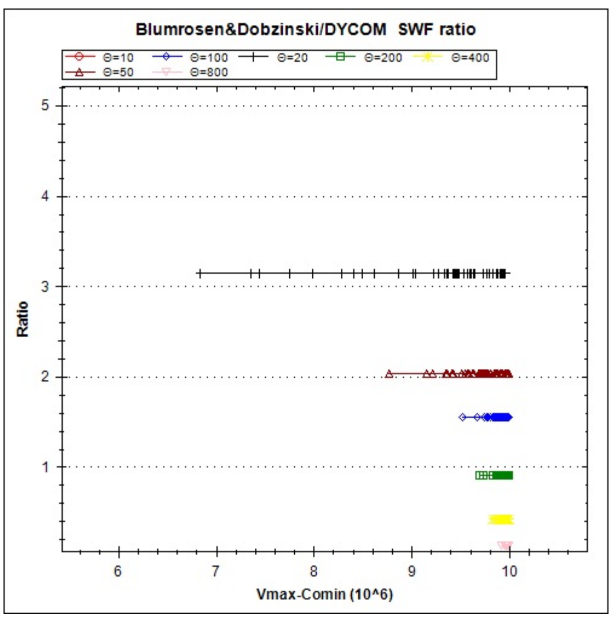

Figure 6: DYCOM's SWF approximation ratio under different bounds of agents' demand/supply level with respect to the overall supply of commodities in the market vs. Blumrosen and Dobzinski 2014 SWF approximation ratio. When demand/supply of each agent is bounded by at least 1/200 of total market units DYCOM's SWF approximation ratio is better than Blumrosen and Dobzinski 2014 even under Blumrosen and Dobzinski limiting assumption of subadditive valuations and costs.

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